New Trends in Mathematical Sciences
http://dx.doi.org/10.20852/ntmsci.2017.128

Partitions of even numbers

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Received: 17 January 2016, Accepted: 8 December 2016
Published online: 23 January 2017.

Abstract: This article presents the concept of the Partitioned Matrix of even numbers $w \geq 4$ and a set of formulas for determining the values of the three possible types of partition: odd composite numbers (C_w), prime numbers or Goldbach partitions (G_w) and partitions of mixed numbers, i.e., a prime plus and odd composite in any order (M_w). The results of the calculated formulas for $10^2 \leq w \leq 10^{22}$ were compared with reference values determined probabilistically as examples. One of the merits of this study is the set of formulas that completes the fundamental equation of the partitions of an even number $w \geq 4$. All the proposed formulas use natural logarithms and are easily calculable. It should be highlighted that the values provided by G_w are averages and are always referred to a set of three sequential even numbers w of types w_2, w_1 and w_0.

Keywords: Partitions of odd composite numbers, mixed partitions, types of even numbers, Goldbach partitions, Goldbach Comet.

1 Introduction

The purpose of this study is to present the concept of the partitioned matrix and the determination of the values of three possible types of partition: odd composite numbers (C_w), prime numbers or Goldbach partitions (G_w) and partitions of mixed numbers, i.e., a prime number plus an odd composite number in any order (M_w). To evaluate the conformity of the proposed formulas for C_w, G_w and M_w, probabilistic principles were used, assuming that the distribution of prime numbers is at least apparently random [5].

The approach, considering the concepts that it uses, is simple, nothing more than an algebraic approach that is sufficient to demonstrate the proposed formulas and the theorem. It is worth remembering the words of Einstein [1], who claimed that most of the fundamental ideas of science are essentially simple and can, as a rule, be expressed in language that can be understood by everyone.

This study is divided into several parts. Initially some symbols and concepts that are used in the article are addressed. Attention is drawn to the concept of the partitioned Matrix for a given even number ≥ 4 and to the typology of even numbers (w_0, w_1, w_2). The assumption that demonstrates the adopted strategy is then presented. In Part 4, partitions of even numbers $w \geq 4$ are discussed. The aim is to demonstrate that the formulas for C_w, G_w and M_w correspond more closely to reality. This reality, however, is far from being empirical, as probabilistic concepts are used to define it, from the Law of Large Numbers [2], reference values. In effect, to be able to affirm that the formulas proposed in this work for C_w, G_w and M_w are applicable and provide values close to reality, it was necessary to "construct" this reality, and this was done using statistic principles. Each calculation, according to the statistical rules (Cw_prob, Gw_prob and Mw_prob) has a clearly illustrated procedure with demonstrative schemes. In Part 5, there is an overview of the formulas for the partitions of even numbers $w \geq 4$. Basically, once again the adaptability of each formula is shown, particularly formulas C_w, G_w and M_w operating together.

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2 Symbols and concepts

Some of the symbols and concepts used in the present study will now be presented.

$\lceil x \rceil$: (i) for any x ε R denotes rounding up

C_w: (i) for any given even number $w \geq 4$, it is the number of partitions made with odd composite numbers irrespective of order, the sum of which is w; ii) e.g., (see Figures 1 and 2) the number $C_{60} = 5$, which means that there are five pairs of odd composite numbers whose sum, irrespective of their order, is 60: {(9;51), (15;45), (21;39), (25;35), (27;33)}; $C_{62} = 1$ {(27;35)} and $C_{64} = 3$, {(9;55), (15;49), (25;39)}; (iii) the number 1, in that it is not prime, is here considered an odd composite number, considering that $1 = 1.1$. Error: (i) is the relative difference between the observed value and the

60

1	1	59
2	3	57
3	5	55
4	7	53
5	**9**	**51**
6	11	49
7	13	47
8	**15**	**45**
9	17	43
10	19	41
11	**21**	**39**
12	23	37
13	**25**	**35**
14	**27**	**33**
15	29	31

C60=	6
G60=	5
M60=	4
L60=	15

62

1	1	61
2	3	59
3	5	57
4	7	55
5	9	53
6	11	51
7	13	49
8	15	47
9	17	45
10	19	43
11	21	41
12	23	39
13	25	37
14	**27**	**35**
15	29	33
16	31	31

C62=	1
G62=	3
M62=	12
L62=	16

64

1	**1**	**63**
2	3	61
3	5	59
4	7	57
5	**9**	**55**
6	11	53
7	13	51
8	**15**	**49**
9	17	47
10	19	45
11	21	43
12	23	41
13	**25**	**39**
14	27	37
15	29	35
16	31	33
17	33	31

C64=	4
G64=	5
M64=	8
L64=	17

Fig. 1: Partitioned matrices of the numbers 60, 62 and 64 as examples.

expected value; (ii) the error is calculated by the difference between the observed value (O) and the expected value (E) divided by the expected value (E), i.e., $error = (O - E)/E$.

G_w: (i) for a given even number $w \geq 4$, this is the number of partitions of prime numbers, irrespective of order, whose sum is w; (ii) is the number of Goldbach partitions; (iii) e.g., (see Figure 1), the number $G_{60} = 6$, which means that there are 6 pairs of prime numbers, the sum of which, irrespective of their order, is 60: {(7.53), (13;47), (17;43), (19;41), (23;37), (29;31)}; $G_{62} = 3$, {(3;59), (19,43), (31,31)} and G_{64}=5, {(3;61), (5;59), (11,53), (17,47), (23;41)}.

L_w: (i) for a given even number $w \geq 4$, it is the number of lines in the Partitioned Matrix; (ii) the value of Lw is given by

$$L_w = \left\lceil \frac{w}{4} \right\rceil \tag{1}$$

(iii) e.g., (see Figure 1) $L_{60} = 15$, $L_{62} = 16$ and $L_{64} = 17$.

Partitioned Matrix of even number $w \geq 4$: (i) matrix composed of two columns and Lw lines showing the possible partitions of odd composite numbers (c_i), prime numbers (g_i) and mixed numbers (m_i), prime plus odd composite or odd composite plus prime related to w; (ii) the first column (Vector A) contains odd numbers arranged in ascending order and the second column (Vector B) has odd numbers arranged in descending order so that the sum of the partitions in each line is w; (iii) the constant values in the column of Vector A are $\leq$ than the constant values in the column of Vector B; (iv) the Partitioned Matrix is said to be structured when the lines are stratified by partitions of odd composite numbers (c_i) and prime numbers (g_i) and mixed numbers (m_i), prime plus odd composite or odd composite plus prime (see example in Figure 2). M_w: (i) for a given even number $w \geq 4$, this is the number of mixed partitions made with an odd composite

	60		
1	**9**	**51**	c1
2	**15**	**45**	c2
3	**21**	**39**	c3
4	**25**	**35**	c4
5	**27**	**33**	c5
6	7	53	g1
7	13	47	g2
8	17	43	g3
9	19	41	g4
10	23	37	g5
11	29	31	g6
12	3	57	m1
13	5	55	m2
14	11	49	m3
15	59	1	m4
	C60=	6	
	G60=	5	
	M60=	4	
	L60=	15	

Fig. 2: Example of structured partitioned matrix, w=60.

number and a prime number, irrespective of order, the sum of which is w; (ii) e.g., (see Figure 1), the number $M_{60} = 4$, which means that there are 4 pairs of composite numbers and prime number whose sum, irrespective of their order, is 60: {(1;59), (3;57), (5;55), (11;49).

P: (i) natural prime number that has exactly two different divisors: the number one is itself; ii) the notion of prime number is reserved only for natural whole numbers [3], by which the present work will consider the set of non-negative whole numbers.

$$N = N^* \cup \{0\} = \{0, 1, 2, 3, ...\}$$

w: (i) even number 2n; ii) there can be three types of even numbers, w_0, w_1, w_2, according to the remainder after dividing by 3; w_0, w_1 and w_2: (ii) w_0 number w type 0 so that $w \mod 3 = 0$; (iii) w_1 number w type 1 so that $w \mod 3 = 1$; (iv) w_2 number w type 2 so that $w \mod 3 = 2$.

π(w): (i) number of prime numbers that exist up to number w; (ii) in this work that studies partitions of numbers $w \geq 4$, strictly speaking, in all counts the prime number 2 should be disregarded; however, as it is irrelevant to the analysis, the count is not adopted as $\pi(w) - 1$; (iii) and the value of $\pi(w)$ considers the approximation proposed by Legendre [4].

$$\pi(w) \approx \frac{w}{LNw - 1.08366} \tag{2}$$

In the calculation of $\pi(w)$ by Formula (2), as shown in Table 1, the error tends to be lower for higher values of w.

w	π(w) real	π(*w*)*Legendre (2)*	*Error*
100	25	28	0.13587631
1000	168	172	0.02202671
10000	1229	1231	0.00123250
100000	9592	9588	-0.00037500
1000000	78498	78543	0.00057553
10000000	664579	665140	0.00084369
100000000	5761455	5768004	0.00113664
1000000000	50847534	50917519	0.00137637
10000000000	455052511	455743004	0.00151739
1E+11	4118054813	4.125E+09	0.00158936
1E+12	37607912018	3.767E+10	0.00161177
1E+13	3.46066E+11	3.466E+11	0.00160536
1E+14	3.20494E+12	3.210E+12	0.00158202
1E+15	2.98446E+13	2.989E+13	0.00154882
1E+16	2.79238E+14	2.797E+14	0.00151015
1E+17	2.62356E+15	2.627E+15	0.00146878
1E+18	2.474E+16	2.478E+16	0.00142643
1E+19	2.34058E+17	2.344E+17	0.00138418
1E+20	2.22082E+18	2.224E+18	0.00134271
1E+21	2.11273E+19	2.115E+19	0.00130242
1E+22	2.01467E+20	2.017E+20	0.00126354

Table 1: Real $\pi(w)$ and Legendre $\pi(w)$ values calculated using Formula (2).

Legend: w: number; $\pi(w)$ real: number of prime numbers up to w clustered empirically; $\pi(w)$ Legendre: number of prime numbers up to number w calculated using Formula (2); Error: error observed between $\pi(w)$ Legendre and $\pi(w)$ real: ($\pi(w)$ Legendre - $\pi(w)$ real)/ $\pi(w)$ real.

3 Partitions of even number ≥ 4

This part is dedicated to showing some lemmas referring to the characteristics of the partitions of even numbers $w \geq 4$. To verify the almost exact correspondence of the proposed formulas in relation to the Cw partitions of odd composite numbers, Gw of prime numbers (or Goldbach partitions) and Mw of mixed partitions (prime and odd composite in any order), reference values were adopted that were calculated probabilistically, considering the apparent randomness of prime numbers [5]. This resource was used, as the real values of C_w, G_w and M_w considered in the tables of example data for $4 \leq w \leq 10^{22}$ were not known.

Lemma 1. *For every even number wgeq4, it is possible to establish the corresponding partitioned matrix demonstration. For every even number $w \geq 4$, the respective set of partitions can be established, with Vector A of the matrix written with odd number t_1 arranged in ascending order and Vector B of the matrix written with odd numbers t_2 arranged in descending order, so that $w = t_1 + t_2$. See Figure 3, which shows as examples the Partitioned Matrices of the even number from 4 to 14; the Partitioned Matrices of the other numbers w are constructed likewise. It should be remembered that the numbers in the column of Vector A are lower than those in Vector B and can be the same in the last line of the matrix.*

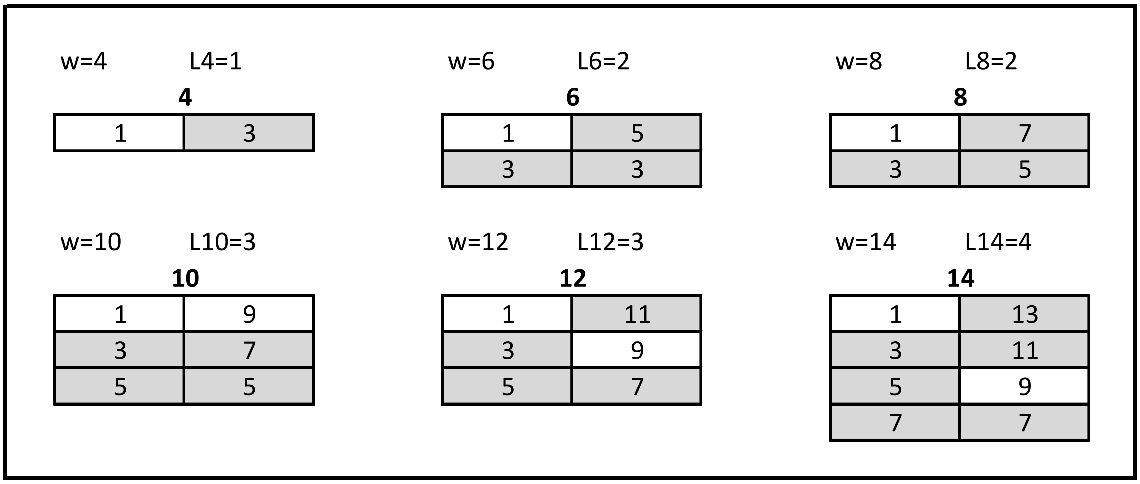

Fig. 3: Example Partition Matrices of even numbers w 4 to 14.

Lemma 2. *For any even number w$\geq$ 4, the Partitioned Matrix has a determined number of lines given by*

$$L_w = \left\lceil \frac{w}{4} \right\rceil .$$

For a given even number $w \geq 4$, the partitions are written as the sum of two odd numbers in such a way that $w = t_1 + t_2$; thus, only half of the numbers $(w/2)$, the odd numbers (t_1 and t_2) are used to establish the partitions of 1 to w. the numbers t_1 and t_2 in turn are distributed in the columns of the Matrix (Vectors A and B), so that the number of lines is $(w/2/2 = w/4)$. As the last line of the Partitioned Matrix of w can be written with the repetition of the number, the number of lines is given by

$$L_w = \left\lceil \frac{w}{4} \right\rceil \tag{3}$$

which corresponds to the division of $w/4$ rounded to the next highest whole when it is not exact.

Lemma 3. *For any even number $w \geq 4$ in the Partitioned Matrix, the number of Lines L_w is equal to the sum of the partitions of odd composite numbers (C_w) with the Goldbach partitions (G_w) and mixed partitions (M_w).*

$$L_w = C_w + G_w + M_w$$

Each line of a Partitioned Matrix is constituted either by two odd composite numbers (c_w) or two prime numbers (g_w); if it is neither of these two cases, the line can only be constituted by an odd composite number and a prime number, in any order, with no other possibility available. Thus:

$$L_w = C_w + G_w + M_w.$$

See example in Figure 1.

Lemma 4. *For any Partitioned Matrix of an even number $w \geq 4$.*

$$G_w = \pi(w) - (L_w - C_w)$$

In any Partitioned Matrix of an even number $w \geq 4$, the number of lines available Lwd to contain the number of prime numbers $\pi(w)$ is given by

$$L_{wd} = (L_w - C_w)\,. \tag{4}$$

Thus, the number of Goldbach partitions (Gw) is given by

$$G_w = \pi(w) - L_{wd} = \pi(w) - (L_w - C_w) \tag{5}$$

$$G_w = \pi(w) - L_w + C_w. \tag{6}$$

This concept can be viewed with the aid of Figure 2: the available Lines L_{wd}, in this case, are those from number 6 to 15, which represents $L_{wd}\ (60) = 10$. Like $\pi\ (60) = 16$, there are more prime numbers than lines available to "receive" them, thus $G(60) = 6$.

One of the non-empirical ways to deduce with a reasonable approximation both the values of Cw and those of Gw is through the concept of probability. Probability Theory is used here only to obtain reference values to check the adequacy of the formulas presented in the present study. According to the Law of Large Numbers [10], if an event of probability p is observed repeatedly, the proportion of the observed frequency of this event in relation to the to the total number of repetition converges towards p as the number of repetitions becomes arbitrarily large. To proceed to the probabilistic deductions in this work, the Structured Partitioned Matrix was considered as shown in Figure 4.

In Figure 4, Vector A of the Structured Partitioned Matrix is divided into two parts: the upper part, designated by A, contains the prime numbers up to $w/2$.

$$A = \pi(\frac{w}{2}) \tag{7}$$

The lower part of Vector A is called Part B and contains the odd composite numbers corresponding to the total lines of the Partitioned Matrix Lw minus the number of prime numbers in Part A;

$$B = L_w - \pi(\frac{w}{2}). \tag{8}$$

In the upper part of Vector B, we have the remaining prime numbers up to w.

$$C = \pi(w) - \pi(\frac{w}{2}). \tag{9}$$

In the lower part of Vector B, we have the remaining odd composite numbers:

$$D = L_w - (\pi(w) - \pi(\frac{w}{2})). \tag{10}$$

The numbers in Part B are considered as being randomly distributed over Parts C and D.

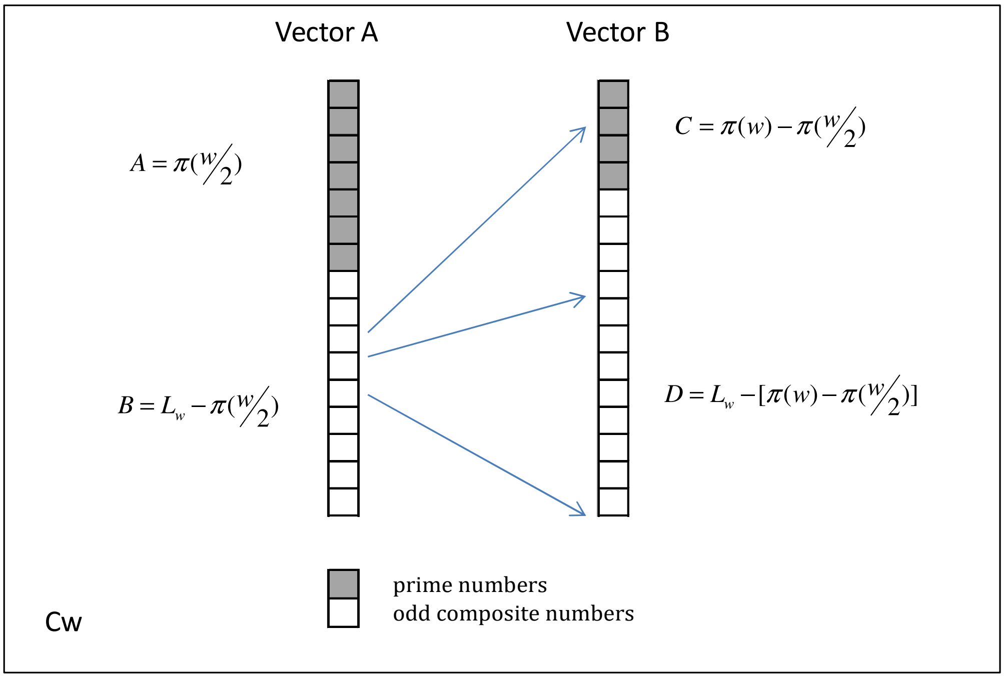

Fig. 4: Layout of the statistical principle for the constitution of partitions of odd composites C_w.

Lemma 5. *The quantity of partitions C_w for any even number $w \geq 4$ in a Partitioned Matrix is probabilistically given by*

$$C_w = (\frac{w}{4} - \pi(\frac{w}{2}))\left(\frac{\frac{w}{4} - \pi(w) + \pi(\frac{w}{2})}{\frac{w}{4}}\right)$$

The purpose of this demonstration based on probability principles is to provide reference values C_w so that other forms of non-probabilistic calculations can be compared. The odd composites from Part B, when joined to the odd composites in Part D (see Figure 4), constitute a partition of odd composite numbers (c_w). The number of C_w calculated probabilistically is expressed by Formula (11).

$$C_w = (L_w - \pi(\frac{w}{2}))\left(\frac{L_w - [\pi(w) - \pi(\frac{w}{2})]}{L_w}\right) \tag{11}$$

$$C_w = B\left(\frac{D}{L_w}\right) \tag{12}$$

$$C_w = (\frac{w}{4} - \pi(\frac{w}{2}))\left(\frac{\frac{w}{4} - \pi(w) + \pi(\frac{w}{2})}{\frac{w}{4}}\right) \tag{13}$$

Table 2, column Cw_prob, shows the reference values of C_w calculated probabilistically according to Formula (11).

Lemma 6. *The Partitioned Matrix of an even number $w \geq 4$ contains a determined number of partitions of odd composite numbers C_w, the value of which is given approximately by*

$$C_w \approx \left(\frac{w}{4}\right)\frac{(LNw - 3,0866)^2}{(LNw - 1,08366)^2}$$

w	π(w)	Lw Rows	π(w) Legendre	Part A: π(w/2)Legendre	Part B (8)	Part C (9)	Part D (10)	Cw_prob.(11)
100	25	25	28	18	7	11	14	4.18
1000	168	250	172	97	153	74	176	107.24
10000	1229	2500	1231	673	1827	558	1942	1419.59
100000	9592	25000	9588	5136	19864	4453	20547	16326.31
1000000	78498	250000	78543	41533	208467	37010	212990	177605.40
10000000	664579	2500000	665140	348644	2151356	316496	2183504	1878998.04
100000000	5761455	25000000	5768004	3004108	21995892	2763895	22236105	19564117.93
1000000000	50847534	250000000	50917519	26390156	223609844	24527363	225472637	201671604.90
10000000000	455052511	2500000000	455743004	235304706	2264695294	220438297	2279561703	2065005063.92
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	2.288E+10	2.002E+09	2.300E+10	2.105E+10
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	2.307E+11	1.833E+10	2.317E+11	2.137E+11
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	2.322E+12	1.690E+11	2.331E+12	2.165E+12
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	2.336E+13	1.568E+12	2.343E+13	2.189E+13
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	2.347E+14	1.463E+13	2.354E+14	2.210E+14
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	2.357E+15	1.371E+14	2.363E+15	2.228E+15
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	2.366E+16	1.289E+15	2.371E+16	2.244E+16
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	2.374E+17	1.217E+16	2.378E+17	2.258E+17
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	2.381E+18	1.153E+17	2.385E+18	2.271E+18
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	2.387E+19	1.094E+18	2.391E+19	2.283E+19
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	2.393E+20	1.042E+19	2.396E+20	2.293E+20
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	2.398E+21	9.943E+19	2.401E+21	2.302E+21

Table 2: Probabilistic calculation of Cw.

Legend: w: number; π(w): number of prime numbers up to w; Lw Rows: number of lines Lw of the Partitioned Matrix; π(w) Legendre: number of prime numbers existing up to w calculated by Formula (2). Part A, Part B, Part C and Part D: values according to Formulas (7) to (10); Cw_prob: value for Cw calculated probabilistically according to Formula (11).

It is demonstrated below that $C_w \approx \left(\frac{w}{4}\right)\frac{(LNw-3,0866)^2}{(LNw-1,08366)^2}$ *adequately expresses the number of partitions of odd composite numbers in a Prioritization Matrix of a given even number* $w \geq 4$*, with this number being very close to the reference value calculated probabilistically. The initial situation is considered as being.*

$$C_w \approx \left(\frac{w}{4}\right)\frac{(LNw-3,0866)^2}{(LNw-1,08366)^2}. \tag{14}$$

Therefore, an equivalent formula can be written

$$C_w \approx \left(\frac{w}{4}\right) - \left(\frac{w}{(LNw-1,08366)} - \frac{w}{(LNw-1,08366)^2}\right) \tag{15}$$

which is the same as

$$C_w \approx \left(\frac{w}{4}\right) - \frac{w}{(LNw-1,08366)} + \frac{w}{(LNw-1,08366)^2}. \tag{16}$$

As $Lw = \left\lceil \frac{w}{4} \right\rceil \ \pi(w) \approx \frac{w}{(LNw-1,08366)}$ *it can be written that.*

$$C_w \approx L_w - \pi(w) + \frac{w}{(LNw-1,08366)^2}. \tag{17}$$

By (6) it is derived that

$$C_w = L_w - \pi(w) + G_w \tag{18}$$

w	π(w)	*Rows*	*π(w) Legendre*	Part A: *π(w/2)Legendre*	*Cw_prob. (12)*	*Cw (14)*	*Error*
100	25	25	28	18	4.18	4.67	0.115790684
1000	168	250	172	97	107.24	107.78	0.005016856
10000	1229	2500	1231	673	1419.59	1420.90	0.000927412
100000	9592	25000	9588	5136	16326.31	16330.97	0.000285420
1000000	78498	250000	78543	41533	177605.40	177625.85	0.000115147
10000000	664579	2500000	665140	348644	1878998.04	1879101.38	0.000055001
100000000	5761455	25000000	5768004	3004108	19564117.93	19564694.96	0.000029494
1000000000	50847534	250000000	50917519	26390156	201671604.90	201675074.89	0.000017206
10000000000	455052511	2500000000	455743004	235304706	2065005063.92	2065027164.93	0.000010703
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	2.105E+10	2.105E+10	0.000007002
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	2.137E+11	2.138E+11	0.000004771
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	2.165E+12	2.165E+12	0.000003362
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	2.189E+13	2.189E+13	0.000002437
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	2.210E+14	2.210E+14	0.000001810
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	2.228E+15	2.228E+15	0.000001372
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	2.244E+16	2.244E+16	0.000001058
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	2.258E+17	2.258E+17	0.000000830
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	2.271E+18	2.271E+18	0.000000660
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	2.283E+19	2.283E+19	0.000000531
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	2.293E+20	2.293E+20	0.000000432
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	2.302E+21	2.302E+21	0.000000355

Table 3: Comparison of values of Cw calculated probabilistically and by Formula (14).

Legend: w: even number w; π(w): number of prime numbers existing up to number w; Rows: number of lines Lw of the Partitioned Matrix; π(w) Legendre: number of prime numbers existing up to number w calculated by Formula (2). Part A: value according to Formula (7); Cw_prob: Value of Cw calculated probabilistically according to Formula (12); Cw: value of Cw calculated by Formula (14); Error: observed error between Cw and Cw prob: (Cw - Cw prob)/ Cw prob.

In this deduction, G_w essentially assumes the value in accordance with Sylvester [9], with the adjustment proposed by Legendre [5]. In effect, in this analysis it is observed that

$$G_w \approx \frac{w}{(LNw - 1.08366)^2}, \tag{19}$$

which, considering the adjustment of Legendre [5], expressed the formula of Sylvester [7]

$$G_w \approx \frac{w}{LNw^2}. \tag{20}$$

Another formula for Gw will be seen later with greater accuracy than Formula (19).

Formula (14), which expresses the number of partitions of odd composite numbers C_w for a given even number $w \geq 4$ adheres significantly to the reference values Cw_prob calculated using probabilistic techniques, as shown in Table 3. It can be said that Formula (14) is adequate for providing the value of C_w in a Partitioned Matrix of a given even number $w \geq 4$. Formula (14) was exemplified in Table 3 with calculations whose values w range from 10^2 to 10^{22}. Considering the probabilistically calculated values as real values or very close to real values, for Formula (14) of C_w the deviations of two values that occurred were calculated. These deviations (errors) tended to shift towards zero when $w \to \infty$ (See Error column in Table 3).

Lemma 7. *Number of partitions of prime numbers G_w for each column $w \geq 4$ in a Partitioned Matrix is given probabilistically by*

$$G_w = \pi(\frac{w}{2}) \left(\frac{\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}} \right)$$

The aim of this demonstration based on probability principles is to provide reference values of Gw so that other forms of non-probabilistic calculation can be compared. The prime numbers in Part A, when joined with the prime numbers in part C (see Figure 5) constitute a partition of prime numbers of Goldbach partition (G_w). The number of Gw calculated probabilistically is expressed by Formula (21).

$$G_w = A\left(\frac{C}{L_w}\right) \tag{21}$$

which is equivalent to

$$G_w = \pi(\frac{w}{2}) \left(\frac{\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}} \right) \tag{22}$$

The numbers in Part A are considered as being randomly distributed over Parts C and D. In Table 4, column Gw_prob(22)

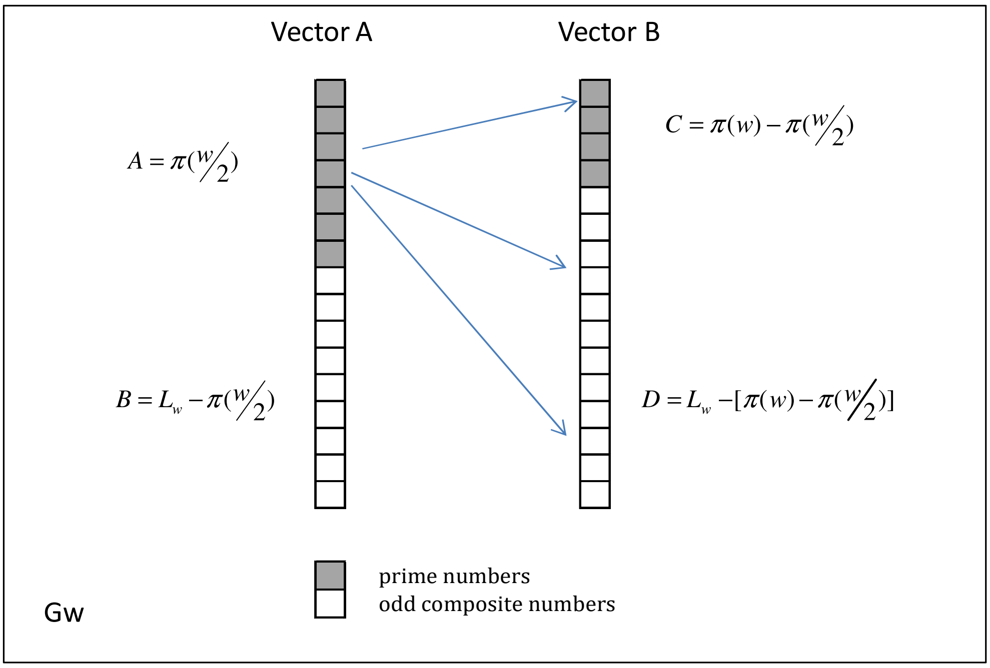

Fig. 5: Diagram of the statistical principle for the constitution of partitions of prime numbers G_w.

shows the reference values of Gw calculated probabilistically according to Formula (22).

Lemma 8. *The Partitioned Matrix of an even number w$\geq$ 4 contains a determined number of partitions of prime numbers G_w, the value of which is given approximately by*

$$G_w \approx w \left(\frac{[2 * (LN\frac{w}{2} - 1.08366)] - (LNw - 1.08366)}{[LNw - 1.08366]\,(LN\frac{w}{2} - 1,08366)^2} \right)$$

w	π(w)	Rows	π(w) Legendre	Part A: π(w/2)Legendre	Part B (8)	Part C (9)	Part D (10)	Gw_prob. (22)
100	25	25	28	18	7	11	14	7.58
1000	168	250	172	97	153	74	176	28.94
10000	1229	2500	1231	673	1827	558	1942	150.10
100000	9592	25000	9588	5136	19864	4453	20547	914.71
1000000	78498	250000	78543	41533	208467	37010	212990	6148.58
10000000	664579	2500000	665140	348644	2151356	316496	2183504	44137.73
100000000	5761455	25000000	5768004	3004108	21995892	2763895	22236105	332121.65
1000000000	50847534	250000000	50917519	26390156	223609844	24527363	225472637	2589123.73
10000000000	455052511	2500000000	455743004	235304706	2264695294	220438297	2279561703	20748067.52
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	2.288E+10	2.002E+09	2.300E+10	1.700E+08
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	2.307E+11	1.833E+10	2.317E+11	1.418E+09
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	2.322E+12	1.690E+11	2.331E+12	1.201E+10
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	2.336E+13	1.568E+12	2.343E+13	1.030E+11
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	2.347E+14	1.463E+13	2.354E+14	8.931E+11
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	2.357E+15	1.371E+14	2.363E+15	7.818E+12
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	2.366E+16	1.289E+15	2.371E+16	6.901E+13
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	2.374E+17	1.217E+16	2.378E+17	6.136E+14
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	2.381E+18	1.153E+17	2.385E+18	5.492E+15
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	2.387E+19	1.094E+18	2.391E+19	4.944E+16
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	2.393E+20	1.042E+19	2.396E+20	4.474E+17
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	2.398E+21	9.943E+19	2.401E+21	4.068E+18

Table 4: Calculation of Gw probabilistically using Formula (22).

Legend: w: even number w; π(w): number of prime numbers up to number w; Rows: number of lines Lw in the Partitioned Matrix; π(w) Legendre: number of prime numbers existing up to number w calculated by Formula (2). Part A to Part D: values according to Formulas (7) to (10); Gw_prob: value of Gw calculated probabilistically according to Formula (22).

It is demonstrated below that

$$G_w \approx w\left(\frac{[2*(LN\frac{w}{2}-1.08366)]-(LNw-1.08366)}{[LNw-1.08366]\,(LN\frac{w}{2}-1,08366)^2}\right) \tag{23}$$

adequately expresses the number of partitions of prime numbers in a Prioritization Matrix of a given even number $w \geq 4$, with this number very close to the reference value calculated probabilistically. Formula (23) was shown to be more accurate than Formula (19).

It can be written, derived from Formula (23)

$$G_w \approx \frac{\frac{w}{2}\{[w(LN\frac{w}{2}-1.08366)]-\frac{w}{2}(LNw-1.08366)\}}{\frac{w}{4}[LNw-1.08366]\,(LN\frac{w}{2}-1,08366)^2} \tag{24}$$

$$G_w \approx \frac{2\{[w(LN\frac{w}{2}-1.08366)]-\frac{w}{2}(LNw-1.08366)\}}{[LNw-1.08366]\,(LN\frac{w}{2}-1,08366)^2} \tag{25}$$

An equivalent formula to this is

$$G_w \approx \left(\frac{\frac{w}{2}}{(LN\frac{w}{2}-1,08366)}\right)\left(\frac{w}{LNw-1.08366}-\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}\right)/\frac{w}{4} \tag{26}$$

$$G_w \approx \left(\frac{\frac{w^2}{2}}{[LN\frac{w}{2}-1,08366](LNw-1.08366)}-\left(\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}\right)^2\right)/\frac{w}{4} \tag{27}$$

$$G_w \approx \left(\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}\right)\left(\frac{w}{(LNw-1.08366)}-\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}\right)/\frac{w}{4}. \tag{28}$$

Expressed otherwise.

$$G_w = \pi(\frac{w}{2})\left(\frac{\pi(w)-\pi(\frac{w}{2})}{\frac{w}{4}}\right) \tag{29}$$

As Equation (29) is equal to Equation (22), it is proved that Equation (23) is adequate for quantifying G_w.

Formula (23), which expresses the number of partitions of prime numbers for a given even number $w \geq 4$, adheres significantly to the reference values for G_w calculated using probabilistic techniques as shown in Table 5. It can be stated that Formula (23) is adequate for supplying the value of G_w in a Partitioned Matrix of a given even number $w \geq 4$.

An example of Formula (23) was shown in Table 5 with calculations whose values of w range from 10^2 to 10^{22}. With the probabilistically calculated values considered real or very close to real values, for Formula (23) of G_w the deviations that occurred between the two values were calculated. These deviations (errors) tended to shift towards zero when $w \to \infty$ (see Error column in Table 5).

w	π(w)	Rows	π(w) Legendre	Part A: π(w/2)Legendre	Part C (9)	Gw_prob. (22)	Gw (23)	Error
100	25	25	28	18	11	7.58	7.58	0.000000000000
1000	168	250	172	97	74	28.00	28.94	0.033679896270
10000	1229	2500	1231	673	558	150.00	150.10	0.000667421238
100000	9592	25000	9588	5136	4453	914.00	914.71	0.000782130662
1000000	78498	250000	78543	41533	37010	6148.00	6148.58	0.000094346325
10000000	664579	2500000	665140	348644	316496	44137.00	44137.73	0.000016636972
100000000	5761455	25000000	5768004	3004108	2763895	332121.00	332121.65	0.000001942416
1000000000	50847534	250000000	50917519	26390156	24527363	2589123.00	2589123.73	0.000000280079
10000000000	455052511	2500000000	455743004	235304706	220438297	20748067.00	20748067.52	0.000000025115
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	2.002E+09	1.700E+08	1.700E+08	0.000000000711
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	1.833E+10	1.418E+09	1.418E+09	0.000000000679
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	1.690E+11	1.201E+10	1.201E+10	0.000000000000
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	1.568E+12	1.030E+11	1.030E+11	0.000000000006
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	1.463E+13	8.931E+11	8.931E+11	0.000000000001
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	1.371E+14	7.818E+12	7.818E+12	0.000000000000
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	1.289E+15	6.901E+13	6.901E+13	0.000000000000
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	1.217E+16	6.136E+14	6.136E+14	0.000000000000
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	1.153E+17	5.492E+15	5.492E+15	0.000000000000
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	1.094E+18	4.944E+16	4.944E+16	0.000000000000
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	1.042E+19	4.474E+17	4.474E+17	0.000000000000
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	9.943E+19	4.068E+18	4.068E+18	0.000000000000

Table 5: Comparison between values of G_w calculated probabilistically and by Formula (23).

Legend: w: even number w; $\pi(w)$: number of prime numbers up to number w; Rows: number of Lines Lw of the Partitioned Matrix; $\pi(w)$ Legendre: number of prime numbers up to number w calculated by Formula (2). Part A and Part C: values according to Formulas (7) and (9); Gw_prob. Value of Cw calculated probabilistically according to Formula (22); Gw: value of Gw calculated by Formula (23); Error: error observed between Gw and Gw prob: (Gw - Gw_ prob)/Gw_ prob.

Lemma 9.*The value of G_w using Formula (23)*

$$G_w \approx w\left(\frac{[2*(LN\frac{w}{2}-1.08366)]-(LNw-1.08366)}{[LNw-1.08366](LN\frac{w}{2}-1,08366)^2}\right)$$

is an adequate average value. The demonstration is empirical. It has already been mentioned that there can be three types of even numbers w, w_0, w_1, w_2 according to the remainder after dividing 3. Table 6 shows the real values and *calculated values of G_w for 9860 $\leq w \leq 9996$. In Table 6, the numbers w are arranged in 3 columns. w_2 for type 2 numbers that can be written in the form w=6x+2; w_1 for type 1 numbers that can be written in the form $w = 6x-2$; and w_0 for type 0 numbers that can be written in the form $w = 6x$. For each of these numbers, the real values of Goldbach partitions or G_w partitions were determined. The average Gw column shows the average observed for the three values G(w2), G(w1) and G(w0). It should be noted that there is a significant difference between the number of Goldbach partitions for the type of number w. In the last line of columns G(w2), G(w1) and G(w0), the averages of the G_w values can be seen. The even numbers of type w_2 and w_1, in the examples in Table 6, have an average close to $G_w = 112$, while the average for type w_0 is more than double. $G_w = 227$. In Table 6, the average Gw shows the observed average for the*

w2	w1	w0	G(w2)	G(w1)	G(w0)	average Gw	Gw (23)	Error
w(6x+2)	w(6x-2)	w(6x)	real	real	real	average real	model average	
9860	9862	9864	144	104	204	**150.67**	**148.51**	-0.0143233
9866	9868	9870	86	99	316	**167.00**	**148.58**	-0.1103178
9872	9874	9876	102	104	208	**138.00**	**148.65**	0.0771392
9878	9880	9882	110	156	200	**155.33**	**148.71**	-0.0426170
9884	9886	9888	117	101	196	**138.00**	**148.78**	0.0781288
9890	9892	9894	146	103	214	**154.33**	**148.85**	-0.0355289
9896	9898	9900	96	118	301	**171.67**	**148.92**	-0.1325148
9902	9904	9906	98	102	211	**137.00**	**148.99**	0.0874931
9908	9910	9912	94	134	233	**153.67**	**149.05**	-0.0300122
9914	9916	9918	100	109	223	**144.00**	**149.12**	0.0355766
9920	9922	9924	141	112	200	**151.00**	**149.19**	-0.0119785
9926	9928	9930	122	108	266	**165.33**	**149.26**	-0.0972210
9932	9934	9936	105	103	202	**136.67**	**149.33**	0.0926415
9938	9940	9942	103	162	200	**155.00**	**149.40**	-0.0361556
9944	9946	9948	113	113	196	**140.67**	**149.46**	0.0625408
9950	9952	9954	126	95	248	**156.33**	**149.53**	-0.0435037
9956	9958	9960	98	105	269	**157.33**	**149.60**	-0.0491498
9962	9964	9966	113	99	217	**143.00**	**149.67**	0.0466335
9968	9970	9972	120	139	194	**151.00**	**149.74**	-0.0083659
9974	9970	9978	93	104	195	**130.67**	**149.80**	0.1464661
9980	9976	9984	136	135	211	**160.67**	**149.87**	-0.0671803
9986	9982	9990	103	110	269	**160.67**	**149.94**	-0.0667562
9992	9988	9996	102	98	255	**151.67**	**150.01**	-0.0109277
		average column	**111.65**	**113.61**	**227.30**	**150.86**	**149.26**	**-0.0056493**

Table 6: Real Gw values and values calculated using Formula (23).

wi: numbers w aligned by type; real: real number of partitions for numbers wi. average Gw: average of the three values of G(wi); Formula (23): value of Gw calculated using Formula (23); Error: error between the calculated value and the observed value = (Gw calculated - Gw real)/ Gw real.

three values G(w2), G(w1) and G(w0) of each line. For instance, for the number 9860, 9862 and 9864, the number of Goldbach partitions counted respectively was 144, 104 and 204, which results in an average of 150.67 partitions. The value calculated by Formula (23) is close, at 148.53.

The last line of Table 6 shows that the observed average of G_w was 150.86 and the average Gw calculated by Formula (23) was 149.27, an error of -0.005577. The column marked error in Table 5 shows that the error tends to shift towards zero as w grows.

Table 7 shows, in the column marked stochastic line, the position of the calculated average, using the minimum and maximum points of G_w as a reference. For example, for the case of the first line of Table 7, the minimum value of G(w *min)=104, the maximum values of G(w max)=204 and the average value calculated using Formula (23) G(w)=148.53. The stochastic value corresponds to 0.45, i.e., the calculated average is at 0.45 between the minimum and maximum values. It should be noted that the average of the stochastic values in the examples in Table 7, as shown in the last line, is 0.33, i.e., the average value calculated by Formula (23) is 1/3 of the difference between the minimum and maximum values. Figure 6 presents Goldbach's Comet [8], stratified by three types of numbers w. It is verified that numbers w_0*

w2	w1	w0	G(w2)	G(w1)	G(w0)	average Gw	Gw (23)	min	max	stochastic line
w(6x+2)	w(6x-2)	w(6x)	real	real	real	average real	model average			of model average
9860	9862	9864	144	104	204	150.67	148.53	104.00	204.00	**0.45**
9866	9868	9870	86	99	316	167.00	148.60	86.00	316.00	**0.27**
9872	9874	9876	102	104	208	138.00	148.67	102.00	208.00	**0.44**
9878	9880	9882	110	156	200	155.33	148.74	110.00	200.00	**0.43**
9884	9886	9888	117	101	196	138.00	148.80	101.00	196.00	**0.50**
9890	9892	9894	146	103	214	154.33	148.87	103.00	214.00	**0.41**
9896	9898	9900	96	118	301	171.67	148.94	96.00	301.00	**0.26**
9902	9904	9906	98	102	211	137.00	149.01	98.00	211.00	**0.45**
9908	9910	9912	94	134	233	153.67	149.08	94.00	233.00	**0.40**
9914	9916	9918	100	109	223	144.00	149.15	100.00	223.00	**0.40**
9920	9922	9924	141	112	200	151.00	149.21	112.00	200.00	**0.42**
9926	9928	9930	122	108	266	165.33	149.28	108.00	266.00	**0.26**
9932	9934	9936	105	103	202	136.67	149.35	103.00	202.00	**0.47**
9938	9940	9942	103	162	200	155.00	149.42	103.00	200.00	**0.48**
9944	9946	9948	113	113	196	140.67	149.49	113.00	196.00	**0.44**
9950	9952	9954	126	95	248	156.33	149.55	95.00	248.00	**0.36**
9956	9958	9960	98	105	269	157.33	149.62	98.00	269.00	**0.30**
9962	9964	9966	113	99	217	143.00	149.69	99.00	217.00	**0.43**
9968	9970	9972	120	139	194	151.00	149.76	120.00	194.00	**0.40**
9974	9970	9978	93	104	195	130.67	149.76	93.00	195.00	**0.56**
9980	9976	9984	136	135	211	160.67	149.83	135.00	211.00	**0.20**
9986	9982	9990	103	110	269	160.67	149.90	103.00	269.00	**0.28**
9992	9988	9996	102	98	255	151.67	149.96	98.00	255.00	**0.33**
		average column	**111.65**	**113.61**	**227.30**	**150.86**	**149.27**	**111.65**	**227.30**	**0.33**

Table 7: Stochastic line of the average values of Formula (23).

wi: numbers w aligned by type; G(wi) real: real number of partitions for numbers wi. average Gw: average of three values G(wi); Gw (23): value of Gw calculated according to Formula (23); min: real minimum value observed in G(w2), G(w1) and G(w0); max: real maximum value observed in G(w2), G(w1) and G(w0); stochastic line: value calculated by (Gw(23)-min)/(max-min).

produce a higher number of partitions than number types w1 and w_2. Figure 6 also shows the dashed line of the average values calculated using Formula (23). These values generally lie 1/3 of the way between the minimum and maximum values. Thus, it can be calculated that the value of G_w using Formula (23)

$$G_w \approx w\left(\frac{[2*(LN\frac{w}{2}-1.08366)]-(LNw-1.08366)}{[LNw-1.08366](LN\frac{w}{2}-1,08366)^2}\right)$$

is an adequate average value that is much closer to the real average value.

Lemma 10. *A demonstration of Goldbach's Conjecture is only necessary from the even number $w \geq 98$. It is known, through (6), that*

$$G_w = \pi(w) - L_w + C_w$$

Assuming the partitions of odd composite numbers C_w=0, we have

$$G_w = \pi(w) - L_w \tag{30}$$

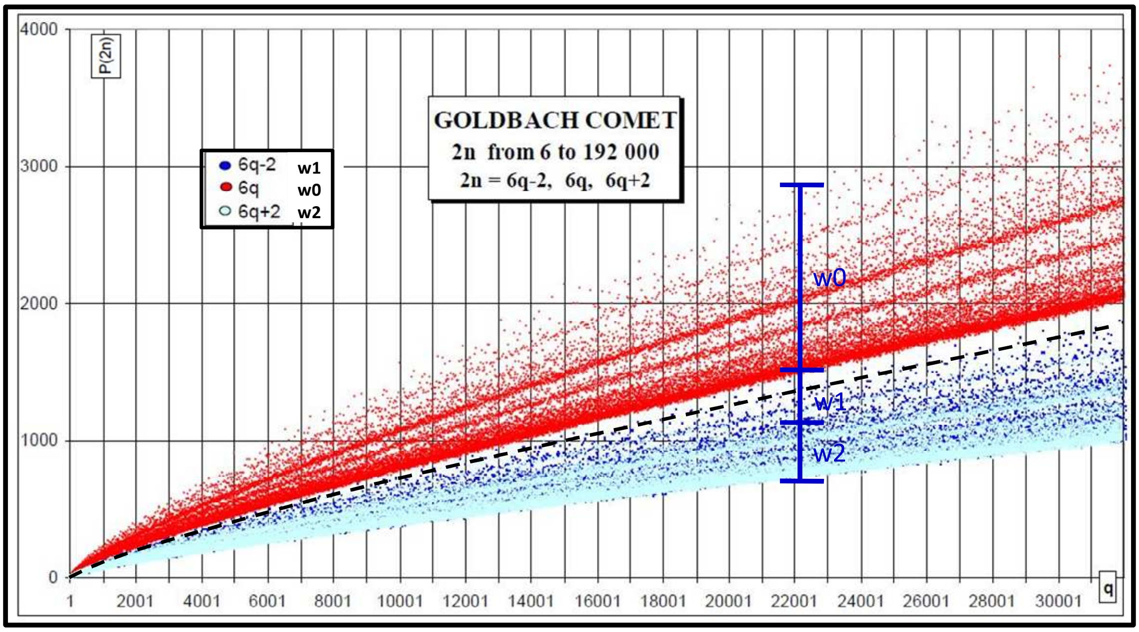

Fig. 6: Goldbach's Comet for even numbers of types w_1, w_0 and w_2. Source: [7] adapted.

Thus, the possibility of Goldbach's Conjecture not being achieved occurs from even number w=98, when the number of $\pi(w)$ for the first time is lower than the number of lines L_w of the Partitioned Matrix, as shown in Table 8. In this case, the prime number 2 is not considered, as it is not part of any Partitioned Matrix. In other words, the negative logical conditions that tests whether $\pi(w) \geq L_w$ only occurs when $w = 98$: up to this number, for $w \geq 4$ the number of primes $\pi(w)$ is greater than the number of lines of the Partitioned Matrix, which inevitably leads to the existence of Goldbach partitions Gw .

Lemma 11. *The number of partitions of mixed numbers M_w, i.e., partitions of prime numbers and partitions of odd composite numbers in any order for even number w≥ 4 in a Partitioned Matrix, probabilistically, is given by*

$$M_w = \pi(\frac{w}{2})\left(\frac{\frac{w}{4} - [\pi(w) - \pi(\frac{w}{2})]}{\frac{w}{4}}\right) + [\frac{w}{4} - (\pi(\frac{w}{2})]\left(\frac{(\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}}\right)$$

This demonstration is based on probability principles. It seeks to obtain reference values for M_w to enable other forms of non-probabilistic calculations to be compared. The prime numbers in Part A, when joined to the odd composite numbers in Part D (see Figure 7) and the odd composite numbers in Part B, when they are joined to the prime numbers in Part C, constitute a mixed partitions, designated by m_w. The number of M_w calculated probabilistically is expressed by Formula (31):

$$M_w = A\left(\frac{D}{L_w}\right) + B\left(\frac{C}{L_w}\right) \tag{31}$$

which can also be expressed as,

$$M_w = \pi(\frac{w}{2})\left(\frac{\frac{w}{4} - [\pi(w) - \pi(\frac{w}{2})]}{\frac{w}{4}}\right) + [\frac{w}{4} - (\pi(\frac{w}{2})]\left(\frac{(\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}}\right) \tag{32}$$

Table 9, in the column market Mw_prob, shows the reference values of M_w calculated probabilistically according to Formula (31).

p	π(w)	w	Lw	π(w)≥Lw
3	1	4	1	**yes**
5	2	6	2	**yes**
7	3	8	2	**yes**
11	4	12	3	**yes**
13	5	14	4	**yes**
17	6	18	5	**yes**
19	7	20	5	**yes**
23	8	24	6	**yes**
29	9	30	8	**yes**
31	10	32	8	**yes**
37	11	38	10	**yes**
41	12	42	11	**yes**
43	13	44	11	**yes**
47	14	48	12	**yes**
53	15	54	14	**yes**
59	16	60	15	**yes**
61	17	62	16	**yes**
67	18	68	17	**yes**
71	19	72	18	**yes**
73	20	74	19	**yes**
79	21	80	20	**yes**
83	22	84	21	**yes**
89	23	90	23	**yes**
97	**24**	**98**	**25**	**no**
101	25	102	26	no
103	26	104	26	yes
107	27	108	27	yes
109	28	110	28	yes
113	29	114	29	yes
127	30	128	32	no
131	31	132	33	no
137	32	138	35	no

Table 8: Number of primes p and number of lines L_w of the Partitioned Matrix.

p: prime number; $\pi(w)$: number of primes lower than w (not counting the number 2); w: even number immediately above the highest prime in column p, corresponding to $p+1$; Lw : number of lines Lw of the Partitioned Matrix, corresponding to number w; $\pi(w) \geq Lw$: logical response (yes/no) that tests the condition $\pi(w) \geq L_w$.

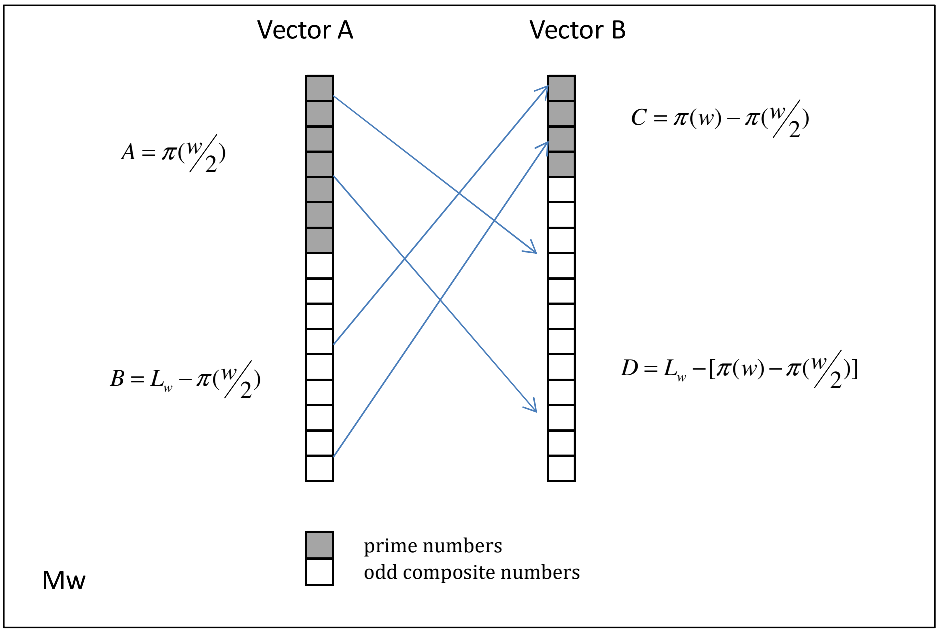

Fig. 7: Layout of the statistical principle for the constitution of multiple partitions M_w (prime and odd composite in any order).

Lemma 12. *The Partitioned Matrix of an even number $w \geq 4$ contains a determined number of partitions of mixed numbers M_w, the value of which is approximately given by*

$$M_w \approx \left(\frac{w}{(LNw - 1{,}08366)}\right)\left(1 - \frac{2}{LNw - 1.08366}\right)$$

w	π(w)	*Rows*	*π(w) Legendre*	Part A: *π(w/2)Legendre*	*Part B (8)*	*Part C (9)*	*Part D (10)*	*Mw_prob. (31)*
100	25	25	28	18	7	11	14	13.24
1000	168	250	172	97	153	74	176	113.81
10000	1229	2500	1231	673	1827	558	1942	930.31
100000	9592	25000	9588	5136	19864	4453	20547	7758.97
1000000	78498	250000	78543	41533	208467	37010	212990	66246.02
10000000	664579	2500000	665140	348644	2151356	316496	2183504	576864.23
100000000	5761455	25000000	5768004	3004108	21995892	2763895	22236105	5103760.42
1000000000	50847534	250000000	50917519	26390156	223609844	24527363	225472637	45739271.38
10000000000	455052511	2500000000	455743004	235304706	2264695294	220438297	2279561703	414246868.56
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	2.288E+10	2.002E+09	2.300E+10	3.785E+09
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	2.307E+11	1.833E+10	2.317E+11	3.483E+10
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	2.322E+12	1.690E+11	2.331E+12	3.226E+11
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	2.336E+13	1.568E+12	2.343E+13	3.004E+12
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	2.347E+14	1.463E+13	2.354E+14	2.810E+13
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	2.357E+15	1.371E+14	2.363E+15	2.640E+14
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	2.366E+16	1.289E+15	2.371E+16	2.489E+15
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	2.374E+17	1.217E+16	2.378E+17	2.355E+16
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	2.381E+18	1.153E+17	2.385E+18	2.234E+17
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	2.387E+19	1.094E+18	2.391E+19	2.125E+18
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	2.393E+20	1.042E+19	2.396E+20	2.026E+19
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	2.398E+21	9.943E+19	2.401E+21	1.936E+20

Table 9: Calculation of M_w probabilistically using Formula (31).

Legend: w: even number w; $\pi(w)$: number of prime numbers up to number w; Rows: number of lines Lw of the Partitioned Matrix; $\pi(w)$ Legendre: number of prime numbers up to number w calculated using Formula (2). Part A, Part B, Part C, Part D: values according to Formulas (7) to (10); Mw_ prob: value of Mw calculated probabilistically according to Formula (31).

It is demonstrated below that

$$M_w \approx \left(\frac{w}{(LNw-1,08366)}\right)\left(1-\frac{2}{LNw-1.08366}\right) \tag{33}$$

adequately expresses the number of mixed partitions M_w in a Prioritization Matrix of a given even number w≥ 4, with this number very close to the value calculated probabilistically by Formula (31). It can be written, derived from Formula (33), that

$$M_w \approx \frac{w}{(LNw-1,08366)} - \frac{2w}{(LNw-1,08366)^2} \tag{34}$$

This formula is equivalent to

$$M_w \approx \left(\frac{\frac{w}{2}}{(LN\frac{w}{2}-1,08366)}\right)\left(\frac{\frac{w}{4}-\left(\frac{w}{LNw-1.08366)}-\frac{w/2}{LN\frac{w}{2}-1.08366)}\right)}{w/4}\right)+\left(\frac{w}{4}-\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}\right)\left(\frac{\frac{w}{LNw-1.08366}-\frac{\frac{w}{2}}{LN\frac{w}{2}-1.08366}}{\frac{w}{4}}\right) \tag{35}$$

which is the same as

$$M_w = \pi(\frac{w}{2})\left(\frac{\frac{w}{4}-[\pi(w)-\pi(\frac{w}{2})]}{\frac{w}{4}}\right)+[\frac{w}{4}-(\pi(\frac{w}{2})]\left(\frac{(\pi(w)-\pi(\frac{w}{2})}{\frac{w}{4}}\right) \tag{36}$$

Equation (36) is equal to Equation (32), which proves the adequacy of Formula (33).

Formula (33), which expresses the number of partitions of mixed numbers Mw for a given even number par $w \geq 4$ adheres significantly to the reference values M_w calculated using probabilistic techniques, as shown in Table 10. It can be said that Formula (33) is adequate for providing the value of M_w in a Partitioned Matrix of a given even number $w \geq 4$. An example of Formula (33) was given in Table 10, the values w of which range from 10^2 to 10^{22}. Considering that the values calculated probabilistically are real or very close to real values, for Formula (33) of G_w the deviations that occurred between the two values were calculated. These errors tend to shift towards zero when $w \to \infty$ (see Errors column in Table 10).

w	π(w)	*Rows*	$\pi(w)$ Legendre	Part A: $\pi(w/2)$Legendre	*Mw_prob. (31)*	*Mw (33)*	*Error*
100	25	25	28	18	13.24	12.27	-0.07317043
1000	168	250	172	97	113.81	112.74	-0.00945435
10000	1229	2500	1231	673	930.31	927.68	-0.00283031
100000	9592	25000	9588	5136	7758.97	7749.65	-0.00120115
1000000	78498	250000	78543	41533	66246.02	66205.12	-0.00061742
10000000	664579	2500000	665140	348644	576864.23	576657.53	-0.00035831
100000000	5761455	25000000	5768004	3004108	5103760.42	5102606.38	-0.00022612
1000000000	50847534	250000000	50917519	26390156	45739271.38	45732331.38	-0.00015173
10000000000	455052511	2500000000	455743004	235304706	414246868.56	414202666.53	-0.00010670
1.000E+11	4.118E+09	2.500E+10	4.125E+09	2.123E+09	3.785E+09	3.784E+09	-0.00007787
1.000E+12	3.761E+10	2.500E+11	3.767E+10	1.934E+10	3.483E+10	3.483E+10	-0.00005856
1.000E+13	3.461E+11	2.500E+12	3.466E+11	1.776E+11	3.226E+11	3.226E+11	-0.00004514
1.000E+14	3.205E+12	2.500E+13	3.210E+12	1.642E+12	3.004E+12	3.004E+12	-0.00003553
1.000E+15	2.984E+13	2.500E+14	2.989E+13	1.526E+13	2.810E+13	2.810E+13	-0.00002846
1.000E+16	2.792E+14	2.500E+15	2.797E+14	1.426E+14	2.640E+14	2.640E+14	-0.00002315
1.000E+17	2.624E+15	2.500E+16	2.627E+15	1.338E+15	2.489E+15	2.489E+15	-0.00001908
1.000E+18	2.474E+16	2.500E+17	2.478E+16	1.260E+16	2.355E+16	2.355E+16	-0.00001592
1.000E+19	2.341E+17	2.500E+18	2.344E+17	1.191E+17	2.234E+17	2.234E+17	-0.00001341
1.000E+20	2.221E+18	2.500E+19	2.224E+18	1.129E+18	2.125E+18	2.125E+18	-0.00001141
1.000E+21	2.113E+19	2.500E+20	2.115E+19	1.073E+19	2.026E+19	2.026E+19	-0.00000978
1.000E+22	2.015E+20	2.500E+21	2.017E+20	1.023E+20	1.936E+20	1.936E+20	-0.00000845

Table 10: Comparison of values of M_w calculated probabilistically and by Formula (33).

Legend: w: even number $w; \pi(w)$: number of prime numbers up to number w; Rows: number of lines Lw of the Partitioned Matrix; $\pi(w)$ Legendre: number of prime numbers up to the number w calculated by Formula (2). Part A: values according to Formula (7); Mw_prob. value of Mw calculated probabilistically according to Formula (31); Mw: value of Mw calculated using Formula (33); Error: error observed between Mw and Mw prob: (Mw-Mw prob)/Mw prob.

4 Form of partitions of even numbers ≥ 4

Theorem 1. *The formulas (14), (23) and (33) are suitable for complete L_w partition. Lemma 6, above, demonstrated that for an even number w≥ 4 a determined number of partitions of odd composites C_w was observed, the value of which is given approximately by*

$$C_w \approx \left(\frac{w}{4}\right)\frac{(LNw-3,0866)^2}{(LNw-1,08366)^2}. \tag{37}$$

The demonstration of Lemma 9 showed that the average value of G$_w$ (Goldbach partitions) is given by the formula

$$G_w \approx w\left(\frac{[2*(LN\frac{w}{2}-1.08366)]-(LNw-1.08366)}{[LNw-1.08366](LN\frac{w}{2}-1,08366)^2}\right). \tag{38}$$

Lemma 8 demonstrated that the Partitioned Matrix of an even number $w \geq 4$ contains a determined number of partitions of mixed numbers M_w, the value of which is given by

$$M_w \approx \left(\frac{w}{(LNw-1,08366)}\right)\left(1-\frac{2}{LNw-1.08366}\right). \tag{39}$$

Considering Formula (33), if the above formula is adequate, it should be observed that

$$L_w = C_w + G_w + M_w$$

i.e., if the formulas are adequate, the sum of values obtained by them should be equal to L_w. Table 11 shows that this is observed: the sum of values $C_w + G_w + M_w$, denominated $L_{w'}$, coincides with the expected value in the L_w column. It is

w	π(w)	Lw (Rows)	Cw (14)	Gw (23)	Mw (33)	Lw'=Cw+Gw+Mw	Error
100	25	25	4.67	7.58	12.27	24.52	-0.01937237
1000	168	250	107.78	28.94	112.74	249.46	-0.00215208
10000	1229	2500	1420.90	150.10	927.68	2498.68	-0.00052662
100000	9592	25000	16330.97	914.71	7749.65	24995.34	-0.00018639
1000000	78498	250000	177625.85	6148.58	66205.12	249979.55	-0.00008180
10000000	664579	2500000	1879101.38	44137.73	576657.53	2499896.65	-0.00004134
100000000	5761455	25000000	19564694.96	332121.65	5102606.38	24999422.98	-0.00002308
1000000000	50847534	250000000	201675074.89	2589123.73	45732331.38	249996530.00	-0.00001388
10000000000	455052511	2500000000	2065027164.93	20748067.52	414202666.53	2499977898.99	-0.00000884
1.000E+11	4.118E+09	2.500E+10	2.105E+10	1.700E+08	3.784E+09	2.500E+10	-0.00000589
1.000E+12	3.761E+10	2.500E+11	2.138E+11	1.418E+09	3.483E+10	2.500E+11	-0.00000408
1.000E+13	3.461E+11	2.500E+12	2.165E+12	1.201E+10	3.226E+11	2.500E+12	-0.00000291
1.000E+14	3.205E+12	2.500E+13	2.189E+13	1.030E+11	3.004E+12	2.500E+13	-0.00000213
1.000E+15	2.984E+13	2.500E+14	2.210E+14	8.931E+11	2.810E+13	2.500E+14	-0.00000160
1.000E+16	2.792E+14	2.500E+15	2.228E+15	7.818E+12	2.640E+14	2.500E+15	-0.00000122
1.000E+17	2.624E+15	2.500E+16	2.244E+16	6.901E+13	2.489E+15	2.500E+16	-0.00000095
1.000E+18	2.474E+16	2.500E+17	2.258E+17	6.136E+14	2.355E+16	2.500E+17	-0.00000075
1.000E+19	2.341E+17	2.500E+18	2.271E+18	5.492E+15	2.234E+17	2.500E+18	-0.00000060
1.000E+20	2.221E+18	2.500E+19	2.283E+19	4.944E+16	2.125E+18	2.500E+19	-0.00000048
1.000E+21	2.113E+19	2.500E+20	2.293E+20	4.474E+17	2.026E+19	2.500E+20	-0.00000040
1.000E+22	2.015E+20	2.500E+21	2.302E+21	4.068E+18	1.936E+20	2.500E+21	-0.00000033

Table 11: Calculated $L_{w'}$ and expected L_w.

Legend: w: even number w; $\pi(w)$: number of prime numbers up to number w; Lw Rows: number of lines Lw of the Partitioned Matrix; Cw: partitions of odd composite numbers calculated using Formula (37); Gw: partitions of prime numbers (Goldbach) calculated using Formula (23); Mw: mixed partitions (prime and odd composite in any order) calculated according to Formula (33); $Lw' = Cw + Gw + Mw$: sum of the values of columns Cw, Gw and Mw, which is theoretically equal to Lw. Error: error observed between Lw and Lw': $(Lw' - Lw)/Lw$.

important to discuss specifically the value of G_w. When the number w is sufficiently large, Sylvester [6] affirms that

$$G_w \approx \frac{w}{LNw^2}. \tag{40}$$

Considering the proposal of Legendre [4], the above formula can be considered equivalent to

$$G_w \approx \frac{w}{(LNw - 1.08366)^2}. \tag{41}$$

Thus, it is possible to compare the formula of Sylvester adjusted by Legendre (24) with Formula (23), proposed above. The result can be viewed in Table 12. The comparison column is the reference value G_w calculated probabilistically by Formula (22). The column marked Gw (23) expresses the calculation according to Formula (23), proposed above, and the column marked Gw_Sylv (41) expresses the calculation using the formula of Sylvester [6] with the adjustment of Legendre [4]. The column marked Error (23)/(22) shows the relative error between the formula proposed here (23) and Formula (22); the column marked Error (41)/(22) shows the relative error between Formula (41) and Formula (22). Although slightly more complex, Formula (23) appears to be more adequate. The instrument developed so far, especially

w	π(w)	Rows	Gw_prob.(22)	Gw (23)	Gw_Sylv (19)	Error (23)/(22)	Error (19)/(22)
100	25	25	7.58	7.58	8.06	0.0000000000	0.0638969596
1000	168	250	28.00	28.94	29.48	0.0336798963	0.0528949103
10000	1229	2500	150.00	150.10	151.42	0.0006674212	0.0094443533
100000	9592	25000	914.00	914.71	919.37	0.0007821307	0.0058804332
1000000	78498	250000	6148.00	6148.58	6169.03	0.0000943463	0.0034207471
10000000	664579	2500000	44137.00	44137.73	44241.08	0.0000166370	0.0023581550
100000000	5761455	25000000	332121.00	332121.65	332698.67	0.0000019424	0.0017393305
1000000000	50847534	250000000	2589123.00	2589123.73	2592593.72	0.0000002801	0.0013405017
10000000000	455052511	2500000000	20748067.00	20748067.52	20770168.53	0.0000000251	0.0010652334
1.000E+11	4.118E+09	2.500E+10	1.700E+08	1.700E+08	1.701E+08	0.0000000007	0.0008669334
1.000E+12	3.761E+10	2.500E+11	1.418E+09	1.418E+09	1.419E+09	0.0000000007	0.0007192849
1.000E+13	3.461E+11	2.500E+12	1.201E+10	1.201E+10	1.201E+10	0.0000000000	0.0006063839
1.000E+14	3.205E+12	2.500E+13	1.030E+11	1.030E+11	1.030E+11	0.0000000000	0.0005181239
1.000E+15	2.984E+13	2.500E+14	8.931E+11	8.931E+11	8.935E+11	0.0000000000	0.0004478219
1.000E+16	2.792E+14	2.500E+15	7.818E+12	7.818E+12	7.821E+12	0.0000000000	0.0003909165
1.000E+17	2.624E+15	2.500E+16	6.901E+13	6.901E+13	6.903E+13	0.0000000000	0.0003442078
1.000E+18	2.474E+16	2.500E+17	6.136E+14	6.136E+14	6.138E+14	0.0000000000	0.0003053973
1.000E+19	2.341E+17	2.500E+18	5.492E+15	5.492E+15	5.493E+15	0.0000000000	0.0002727995
1.000E+20	2.221E+18	2.500E+19	4.944E+16	4.944E+16	4.945E+16	0.0000000000	0.0002451559
1.000E+21	2.113E+19	2.500E+20	4.474E+17	4.474E+17	4.475E+17	0.0000000000	0.0002215109
1.000E+22	2.015E+20	2.500E+21	4.068E+18	4.068E+18	4.069E+18	0.0000000000	0.0002011290

Table 12: Comparison of accuracy between Formulas (23) and (41) regarding G_w.

Legend: w: even number w; $\pi(w)$: number of prime numbers up to number w; Rows: number of lines Lw of the Partitioned Matrix; Gw_prob (22): probabilistic value of Gw calculated using Formula (22); Gw (23): value of Gw calculated using Formula (23); Gw_ylv (39): value of Gm calculated according to Formula (19); Error (23)/(22): error observed between Gw(23) and Gw(22): (Gw(23)-Gw(22))/Gw(22); Error (19)/(22): error observed between Gw(41) and Gw(22): (Gw(41) - Gw(22))/ Gw(22).

the concept of the Partitioned Matrix and Formulas (33), (37), (23) and (33),

$$(L_w = C_w + G_w + M_w \tag{42}$$

$$C_w \approx \left(\frac{w}{4}\right)\frac{(LNw-3,0866)^2}{(LNw-1,08366)^2} \tag{43}$$

$$G_w \approx w\left(\frac{[2*(LN\frac{w}{2}-1.08366)]-(LNw-1.08366)}{[LNw-1.08366](LN\frac{w}{2}-1,08366)^2}\right) \tag{44}$$

$$M_w \approx \left(\frac{w}{(LNw-1,08366)}\right)\left(1-\frac{2}{LNw-1.08366}\right) \tag{45}$$

although simple have proved to be adequately accurate. Table 13 shows three extracts of Tables 3, 5 and 10 with selected lines pertaining to w between 10^8 and 10^{10} highlighting the last column (Error) referring to the deviations between the reference values calculated probabilistically (Cw_prob (12), Gw_prob (22) and Mw_prob (31)) and the values calculated by the proposed Formulas (14), (23) and (33). The deviations are in the order of ten thousandths. It can thus be affirmed that the formula proposed here is a logical, easily understandable and robust instrument. The probabilistic values calculated with reference values, as shown in Figures 4, 5 and 7, were the resources used, as the real values of C_w, G_w and M_w were

w	π(w)	Rows	π(w) Legendre	Cw_prob. (12)	Cw (14)	Error
100000000	5761455	25000000	5768004	19564117.93	19564694.96	0.0000294939
1000000000	50847534	250000000	50917519	201671604.90	201675074.89	0.0000172062
10000000000	455052511	2500000000	455743004	2065005063.92	2065027164.93	0.0000107026

w	π(w)	Rows	π(w) Legendre	Gw_prob. (22)	Gw (23)	Error
100000000	5761455	25000000	5768004	332121.00	332121.65	0.0000019424
1000000000	50847534	250000000	50917519	2589123.00	2589123.73	0.0000002801
10000000000	455052511	2500000000	455743004	20748067.00	20748067.52	0.0000000251

w	π(w)	Rows	π(w) Legendre	Mw_prob. (31)	Mw (33)	Error
100000000	5761455	25000000	5768004	5103760.42	5102606.38	-0.0002261168
1000000000	50847534	250000000	50917519	45739271.38	45732331.38	-0.0001517295
10000000000	455052511	2500000000	455743004	414246868.56	414202666.53	-0.0001067045

Table 13: Deviations (errors) for C_w, G_w and M_w for w between 10^8 and 10^{10}.

Legend: w: even number w; $\pi(w)$: number of prime numbers up to number w; Rows: number of lines Lw of the Partitioned Matrix; $\pi(w)$ Legendre: number of prime numbers up to number w calculated using Formula (2). Part A: values according to Formula (7); Cw prob. (12) Gw prob (22) Mw prob (31): reference values calculated respectively by Formulas (12), (22) and (31); Cw (14), Gw (23), Mw(33): values calculated respectively using proposed Formulas (37), (23) and (33); Error: error observed between the columns marked prob and proposed formulas.

unknown. If these values were empirically known, it would be possible to evaluate Formulas (37), (23) and (33) with greater propriety and accuracy.

5 Conclusion and recommendations

The initial concept of this work was the Partitioned Matrix of an even number w$\geq$ 4, and it was shown that for every even number w$\geq$ 4 it is possible to establish a corresponding Partitioned Matrix with a determined number of lines given by

$$L_w = \left\lceil \frac{w}{4} \right\rceil .$$

It was demonstrated that, fundamentally, the sum of the partitions is equal to the number of lines in the matrix: $L_w = C_w + G_w + M_w$.

It was also shown that for each and every Partitioned Matrix of an even number $w \geq 4$ it is observed that $G_w = \pi(w) - (L_w - C_w)$, which means that the number of Goldbach partitions or partitions of prime numbers of an even number $w \geq 4$ is given by the number of prime numbers up to w minus the number of available lines (L_{wd}) calculated as follows: $L_{wd} = L_w - C_w$.

To analyze the adequacy of the proposed formulas, probabilistically calculated reference values were adopted. Figure 4 shows the layout of values for the probabilistic calculation of C_w, G_w and M_w. The formulas used for the reference values were numbers (13), (22) and (32):

$$C_w = (\frac{w}{4} - \pi(\frac{w}{2})) \left(\frac{\frac{w}{4} - \pi(w) + \pi(\frac{w}{2})}{\frac{w}{4}} \right) \tag{46}$$

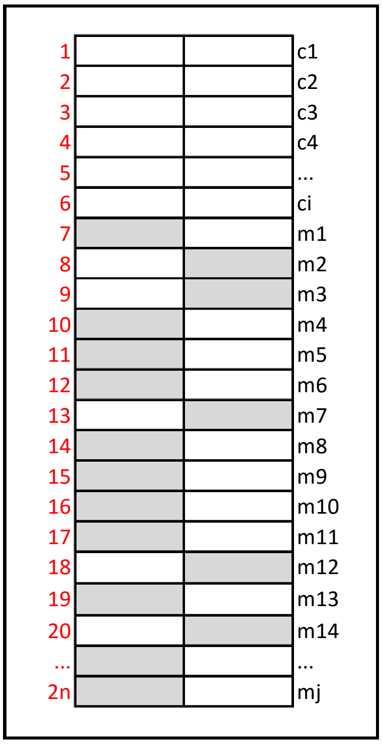

Fig. 8: Example of a structured Partitioned Matrix, $w = 2n$.

$$G_w = \pi(\frac{w}{2}) \left(\frac{\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}} \right) \tag{47}$$

$$M_w = \pi(\frac{w}{2}) \left(\frac{\frac{w}{4} - [\pi(w) - \pi(\frac{w}{2})]}{\frac{w}{4}} \right) + [\frac{w}{4} - (\pi(\frac{w}{2})] \left(\frac{(\pi(w) - \pi(\frac{w}{2})}{\frac{w}{4}} \right) \tag{48}$$

formulas generated the reference values shown in the tables for the values of C_w, G_w and M_w that range from 10^2 to 10^{22}, enabling the verification that the proposed formulas are well adjusted and provide appropriate values, as the deviations between the values of the proposed formulas and the reference values are negligible.

The proposed Formulas (14), (23) and (33) are:

$$C_w \approx \left(\frac{w}{4}\right) \frac{(LNw - 3,0866)^2}{(LNw - 1,08366)^2} \tag{49}$$

$$G_w \approx w \left(\frac{[2 * (LN\frac{w}{2} - 1.08366)] - (LNw - 1.08366)}{[LNw - 1.08366]\,(LN\frac{w}{2} - 1,08366)^2} \right) \tag{50}$$

$$M_w \approx \left(\frac{w}{(LNw - 1,08366)} \right) \left(1 - \frac{2}{LNw - 1.08366} \right). \tag{51}$$

Formula (14) is innovative as it refers to partitions of odd composite numbers (C_w), an important concept within Formula (33) $L_w = C_w + G_w + M_w$. The partitions of odd composite numbers (C_w) are as important as the partitions of prime numbers or Goldbach partitions (G_w). The number of partitions C_w is fundamental for defining the available lines (L_{wd}) in a Partitioned Matrix that explain the existence of partitions G_w or Goldbach partitions.

Formula (23) expresses the number of prime partitions or Goldbach partitions (G_w) and was shown to be more accurate than Formula (19) in accordance with Sylvester [7], with the adjustment proposed by Legendre [5]:

$$G_w \approx \frac{w}{(LNw - 1.08366)^2} \tag{52}$$

Table 12 shows a comparison of the accuracy of Formulas (23) and (19) regarding the G_w and it can be concluded that, although slightly more complex, Formula (23) is more adequate than Formula (19).

Table 6 compares real G_w values and those calculated using Formula (23), and shows that Formula (23) has an average value. In effect, there are three types of even numbers (G_0, G_1 and G_2), according to the remainder after dividing by 3. The number of partitions of G_w is closely associated with the type of number, as shown by Goldbach's Comet (Figure 6). Thus, it should be understood that the value of G_w calculated for a certain even number $w \geq 4$ actually expresses the average value regarding G(w-2), G(w) and G(w+2).

It was also shown, rigorously, that the demonstration of Goldbach's Conjecture is necessary only from the even number w$\geq$ 98, as up to this number the primes $\pi(w)$ is considered higher than the number of lines of the Partitioned Matrix, which inevitably leads to the existence of Goldbach partitions G_w (see Table 8).

Formula (33) is also concerned with a new concept, which is that of mixed partitions (M_w), i.e., partitions of even numbers $w \geq 4$ constituted by a prime number and an odd composite number in any order. Table 10 shows a comparison between the values of M_w calculated probabilistically and with Formula (33), showing that this formula is adequate.

All the proposed formulas C_w, G_w and M_w make use of easily calculable natural logarithms, providing values that are very close to the reference values.

Based on the Law of Great Numbers, this study assumed that reference values can be obtained statistically for C_w, G_w and M_w. The correct method for testing the accuracy of the proposed formulas is to work with real values. Thus, it is recommended that computer science scholars can assist with their studies in order to substitute reference values for real values Cw_prob, Gw_prob and Mw_prob calculated probabilistically and adopted here. Another point has to do with the average value of G_w (as shown in Table 6) and the stochastic position of the average value in relation to the real minimum and maximum values of Gw regarding $G(w-2)$, $G(w)$ and $G(w+2)$. In other words, it is necessary to expand Tables 6 and 7 to have a notion of the true accuracy of the proposed formula and verify whether the average value calculated by Formula (23) is effectively located 1/3 of the way between the maximum and minimum values of $G(w-2)$, $G(w)$ and $G(w+2)$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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